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A STUDY ABOUT MATERIAL POINT RELATIVE MOTION UNDER ACTION OF LINEAR OSCILATORY FORCE

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Abstract. The paper presents a study regarding relative motion of a solid body (rigid) on a planar horizontal surface, driven into rectilinear harmonic oscillatory movement, as a functional principle of an inertial transporting mechanism. For theoretical study it adopts some simplifying hypothesis and results can be extended to practical applications.

Keywords: relative motion; relative equilibrium; sliding friction; inertial transporting mechanism.

1. Introduction

As mechanical model of an inertial transporting mechanism, it proposes a system composed by a planar horizontal surface (P) and a solid (rigid) body (W) sitting on this surface (Fig. 1) (Barbu *et al*., 1998). It considers body, of rectangular parallelepiped shape, but it can be assimilated as a material point, taking into account rectilinear translating motion. Between body and support plane there is sliding friction.

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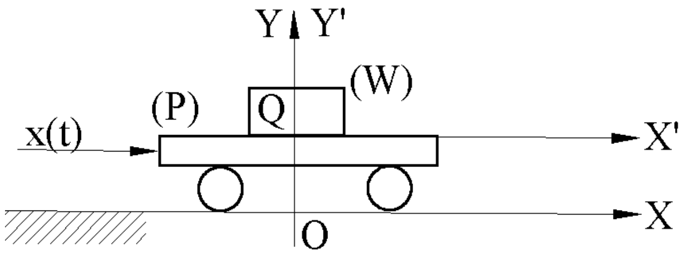

Fig. 1 – Mechanical model of an inertial transporting mechanism.

2. Theoretical Considerations and Aspects

Mechanical model reports to an orthogonal fixed referential system *Oxy* with originated into stable equilibrium position of (*P*) plane and to a mobile system *Ox'y'* with origin posed in plane median point (Grămescu and Domente, 1994). The *Ox* and *Ox'* axes poses on the rectilinear motion direction, *Oy* and *Oy'* are vertical ones.

Mechanical model represents a relative motion problem. The (*P*) plane motion reported to the fixed system is transport motion but body (*W*) motion on the support plane (*P*) is relative motion.

Let consider *m* – body (*W*) mass and μ – sliding friction coefficient.

The support (*P*) plane makes a rectilinear harmonic oscillatory movement to horizontal direction (Levitskii, 1988). It consider motion law,

$$x_Q(t) = A_0 \cdot \sin \omega_0 t\,,\ t \geq 0 \tag{1}$$

where t represents time and A_0 , ω_0 are positive constants, $\langle A_0 \rangle = m$, $\langle \omega_0 \rangle = s^{-1}$.

In order to simplify calculations it considers that plane (*P*) motion takes place without initial phase, *i.e.* plane begins its motion from stable equilibrium position:

$$x_Q(0) = 0\,, \tag{2}$$

At initial moment the two referential systems coincides.

Relative motion of (*W*) body on the (*P*) plane occurs in condition which inertial force is greater than friction force, *i.e.* transport is greater than report between friction force and mass of the body.

In condition when there is relative motion between body (*W*) and plane (*P*), friction force is:

$$F_f = \mu N = \mu m g\,, \tag{3}$$

where, N is normal reaction and g – gravitational acceleration.

So, condition of relative motion of (W) body on the (P) plane can be written as:

$$\left|a_t\right| > \mu g\,. \tag{4}$$

Transport acceleration is:

$$a_t = \frac{d^2 x_Q}{dt^2} = \ddot{x}_Q\,. \tag{5}$$

According to (1) we have:

$$\dot{x}_Q = A_0\omega_0 \cos\omega_0 t\,,\quad \ddot{x}_Q = -A_0\omega_0^2 \sin\omega_0 t\,,\quad \ddot{x}_Q = -\omega_0^2 x_Q\,. \tag{6}$$

Replacing (5) and (6) in (4) it obtains:

$$\omega_0^2\left|x_Q\right| > \mu g\,,\quad \left|x_Q\right| > \frac{\mu g}{\omega_0^2}\,,$$

or, according to (1):

$$\left|\sin\omega_0 t\right| > \frac{\mu g}{\omega_0^2 A_0}\,, \tag{7}$$

This is condition of body (W) relative motion.

Also, existence condition of sine function must be satisfied:

$$\frac{\mu g}{\omega_0^2 A_0} < 1\,,\quad \omega_0^2 A_0 > \mu g\,, \tag{8}$$

relationship between amplitude A_0 and velocity ω_0 of transport motion.

If (8) relation is not satisfied, body (W) will be in state of relative equilibrium, *i.e.* it has not motion reported to the plane.

The transport motion period is

$$T = \frac{2\pi}{\omega_0}\langle s\rangle\,, \tag{9}$$

In hypothesis that condition (8) is fulfilled, the problem occurs to determine the time interval into which there is relative motion.

Writing

$$\frac{\mu g}{\omega_0^2 A_0} = \sin\theta\,,\quad \theta \in \left(0, \frac{\pi}{2}\right), \tag{10}$$

condition (7) becomes,

$$\left|\sin\omega_0 t\right| > \sin\theta\,,\quad t \in \left[0, T\right)\,, \tag{11}$$

representing a trigonometric equation with solution (Fig. 2):

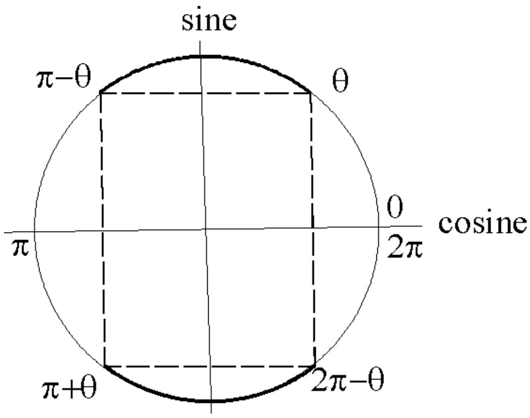

Fig. 2 – Relative motion intervals on trigonometric circle.

$\omega_0 t \in (\theta, \pi-\theta)$ and $\omega_0 t \in (\pi+\theta, 2\pi-\theta)$, or

$$\begin{aligned} & t \in \left(\frac{\theta}{\omega_0}, \frac{\pi}{\omega_0} - \frac{\theta}{\omega_0} \right) \langle s \rangle, \\ & t \in \left(\frac{\pi}{\omega_0} + \frac{\theta}{\omega_0}, \frac{2\pi}{\omega_0} - \frac{\theta}{\omega_0} \right) \langle s \rangle, \end{aligned} \tag{12}$$

were, according to made before notation,

$$\theta = \arcsin \frac{\mu g}{\omega_0^2 A_0} .$$

Times intervals of (12) correspond to relative motion of (*W*) body on the (*P*) plane. These intervals are located in the vicinity of the (*P*) plane maximum elongation moments.

4. Conclusions

1. When the transport motion is oscillatory harmonic, the relative motion is also oscillatory.

2. In order to present the mechanical model to be an inertial transport mechanism, *i.e.* for practical applications, it's needs that the motion of support (P) plane to be periodic.

3. Several practical solutions are available for an inertial transport mechanism, one of them consist of actuating the support plane by the rocker of a four bar mechanism.

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UN STUDIU AL MIŞCĂRII RELATIVE A PUNCTULUI MATERIAL SUB ACŢIUNEA FORŢEI OSCILATORII LINIARE

(Rezumat)

În lucrare se studiază mişcarea relativă a unui corp solid (rigid) pe o suprafaţă plană orizontală, acţionată în mişcare rectilinie oscilatorie armonică, ca principiu de funcţionare al unui mecanism de transport inerţial. Pentru studiul teoretic se adoptă unele ipoteze simplificatoare, iar rezultatele pot fi extinse pentru aplicaţii practice.